An Analytical Model to Predict Bedload Transport in Oscillating Water Tunnels

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1 Motivation

Prediction of nearshore sediment transport is of crucial importance to coastal engineers. Nearshore waves are both asymmetric (forward-leaning in shape) and skewed (with peaked, narrow crests and wide, flat troughs); the effect of such wave shapes on sediment transport is not well understood. Models for predicting sediment transport rates due to nearshore waves are often validated using laboratory data obtained in oscillating water tunnels (OWTs). In an OWT, the oscillatory motion produced by the piston propagates almost instantaneously to the entire tunnel. Consequently, unlike the wave motion in the sea or in a wave flume, flow in an OWT is uniform along the tunnel, and wave propagation effects (such as Longuet-Higgins's streaming) are absent. The effect of these hydrodynamic differences between OWT and sea waves on sediment transport rates has generally been neglected. In this paper we present the results of an analytical study of the hydrodynamics of an OWT, from which we obtain practical formulas to predict OWT bedload transport rates. Our results suggest that the differences in boundary-layer hydrodynamics between OWT and sea waves significantly affect sediment transport rates. Such differences must be accounted for when extrapolating OWT measurements to field conditions.

2 Hydrodynamics of an Oscillating Water Tunnel

We present an analytical characterization of the boundary layer flow in an OWT under an asymmetric and skewed wave (characterized by its two first Fourier harmonics) plus a weak current. Analogous to the work of Trowbridge and Madsen (1984), we account for the time dependence of the eddy viscosity. While Trowbridge and Madsen considered the case of a propagating pure wave, we study a non-propagating wave in an OWT and account for an imposed collinear weak current. Our analysis yields closed-form analytical solutions for the flow field and the bed shear stresses in an OWT. Even in the absence of a current, we identify the existence of a mean flow (boundary layer streaming) that arises from the interaction between the velocity and the time-varying eddy viscosity. Our mean-flow (streaming) predictions agree with experimental data by Ribberink and Al-Salem (1995). This is, to the author's knowledge, the first successful and fully-predictive explanation of Ribberink and Al Salem's streaming profiles.

3 Bedload in Oscillating Water Tunnels

The analytical solutions for bed shear stresses are applied to compute bedload. Following our previous work (Gonzalez-Rodriguez and Madsen, 2008), the instantaneous bedload is predicted using Madsen's (1991) formula, which is similar in form to the empirical Meyer-Peter and Müller formula. Madsen's formula yields the instantaneous transport as a function of the instantaneous bed shear stress, which is provided by the analytical hydrodynamic model. Our analytical expressions of the bed shear stress involve the evaluation of Kelvin functions and the use of complex numbers. To

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simplify the evaluation of these analytical expressions, in this paper we present a practical formulation that approximates the analytical bed shear stress solution. In this approximate formulation, the first, second, and third harmonics of the bed shear stress are evaluated separately using fitted expressions for friction factors and phase shifts, and then added up to obtain the total bed shear stress, i.e.,

$$\tau_b = \tau_{bm,1}\cos(\omega t + \varphi_1) + \tau_{bm,2\alpha}\cos[2(\omega t + \varphi_{2\alpha})] + \tau_{bm,2\beta}\cos[2(\omega t + \varphi_{2\beta})] + \tau_{bm,3}\cos[3(\omega t + \varphi_3)].$$

We also present simplified expressions to compute the current shear stress in the case where an external current is prescribed through a reference current velocity. The model successfully predicts bedload transport rates measured in several experimental studies for skewed waves (as shown in Figure 1), asymmetric waves, and sinusoidal waves plus a current. Unlike the simple conceptual model previously developed by the authors (Gonzalez-Rodriguez and Madsen, 2008), which required and ad-hoc choice of roughness for cases with and without current, the new rigorous analytical model yields consistently good predictions based on the mobile-bed roughness.

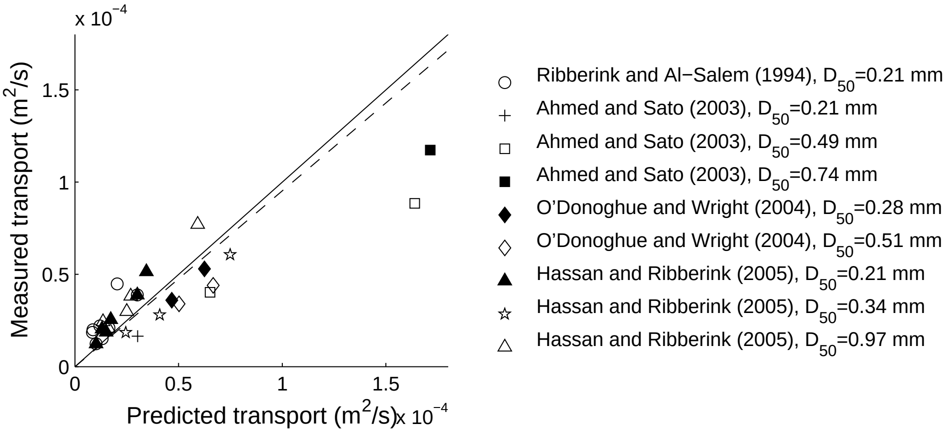

Figure 1: Comparison between measured and predicted average sediment transport rates under skewed, symmetric waves for bedload-dominated cases. The solid line corresponds to perfect agreement between predictions and measurements, while the dashed line is the least-square fit to the data (excluding the two data points with the largest transport rates).

4 Effect of wave propagation

Finally, we discuss the effect of wave propagation on bedload transport rates. By generalizing our analytical model to account for wave propagation, we compare the hydrodynamics and sediment transport rates under propagating waves (in the sea or in a wave flume) and non-propagating waves (in an OWT). This comparison shows significant differences (by a factor of three or more for the cases considered) in the expected net sediment transport rates between a propagating and a non-propagating wave. Therefore, sediment transport rates observed in a wave flume or in the sea could significantly differ from those measured under analogous conditions in an OWT. Such differences must be accounted for when interpreting the results of OWT experiments.

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